Rauzy fractals associated with cubic real number fields

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Abstract

In the early 80’s, G. Rauzy exhibited very nice properties of the so-called Tribonacci number $\alpha$, that is, a root of $x^3 = x^2 + x + 1$. Indeed, he stated that the translation with the vector $(\alpha, \alpha^2)$ over the two-dimensional torus $\mathbb{T}^2$ can be measure-theoretically encoded into the symbolic dynamical system associated to a combinatorial substitution, that is, a morphism of the free monoid.

The main point in the proof of this theorem is to exhibit a relevant fundamental domain for the torus, associated with a suitable partition of this domain such that the toral translation is appropriately encoded by a self-replicating process. The fundamental domain in called the Rauzy fractal. Since this time, a large literature was devoted to the generalization of this result: which toral translations are encoded by a symbolic substitutive system?

In this talk, we prove that any cubic number field $\mathbb{K}$ contains a pair $(\alpha, \beta)$ such that the $\mathbb{T}^2$-translation with vector $(\alpha, \beta)$ is measure-theoretically isomorphic to a substitutive dynamical system.

The proof combines several tools: (1) generation of cubic numbers with the Jacobi-Perron algorithm (following Dubois-Paysan-Le Roux), (2) construction of candidates for a fundamental domain with geometric combinatorial rules introduced by Anroux and Ito, (3) Introduction of slight combinatorial tools to prove that the fundamental domain is relevant.

This work is a common work with Valérie Berthé and Timo Jolivet (Univ. Paris Diderot).