Properties of $N$-expansions

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Abstract

In 2008, Ed Burger and his co-authors showed in [BGKWY] that every quadratic irrational number $x$ has infinitely many $N$-expansions of period length 1. Here, for every integer $N$ different from 1, an $N$-expansion of $x \in [0, 1)$ is a continued fraction expansion of $x$ of the form

$$x = \frac{N}{c_1 + \frac{N}{c_2 + \cdots}};$$

where the partial quotients $c_i$ are integers (which are positive if $N \geq 1$). Note that if $N = 1$ the $N$-expansion is the classical regular continued fraction expansion (RCF).

Last year, Maxwell Anselm and Steven Weintraub showed in [AW] that in fact every $x \in (0, 1)$ has infinitely many $N$-expansions for every $N \geq 2$ fixed. There is one special case, in which the partial quotients $c_i$ are always at least of size $N$. This special case (which Anselm and Weintraub call best $cf_N$-expansions), which can be seen as an analogue of the RCF, has properties which are very similar to those of the RCF, and properties which seem surprisingly different. For example, in this talk I will show that for these best $cf_N$-expansions there is an underlying Gauss-map, and for this map the invariant measure is known. However, Anselm and Weintraub conjecture that there are $N \geq 2$ and quadratic irrationals for which the corresponding best $cf_N$-expansions do not have an ultimately periodic expansion. Further support for this conjecture will be given in this talk.

Finally, a variation of the $N$-expansions will be discussed which yields continued fraction expansions of $x \in (0, 1)$ of the form

$$x = \frac{a_1}{b_1 + \frac{a_2}{b_2 + \cdots}},$$

where the $a_i$ and $b_i$ are positive integers, and $b_i \in \{a_i^2, \ldots, a_i^2 + a_i - 1\}$. The underlying Gauss-map of this expansion will be given. Unfortunately, most properties of this expansion (invariant measure, etc.) are still unknown.
References
