On the geometry of the beta-conjugates of a real algebraic number for the Rényi-Parry numeration system

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Abstract

Let $\beta > 1$ be an algebraic number. Denote by $d_\beta(1) = 0.t_1t_2t_3\ldots$ the Rényi beta-expansion of 1. The geometry of the beta-conjugates of $\beta$ is investigated using the dynamical zeta function $\zeta_\beta(z)$ of the beta-transformation (Ito, Takahashi; Flatto, Lagarias, Poonen; Solomyak; Baladi) and the Parry Upper function $f_\beta(z) = -1 + \sum_{i\geq 1} t_i z^i$. We follow the approach consisting in exhibiting a two variable formal series to make explicit the decomposition of $f_\beta(z)$ as a product, to obtain conditions for a germ of curve. We discuss cases of $\beta$ where no germ appears. If a germ occurs, then the Newton-Puiseux algorithm applied to the Newton polygon provides branches and Puiseux conjugation among beta-conjugates. Inverses of beta-conjugates are important complex numbers attached to $\beta$, in particular since they are poles of the dynamical zeta function if $\beta$ is a Parry number. The geometry of the beta-conjugates located close to the origin of the germ is described by Puiseux series while that of the others, off the origin of the germ, may arise from the cancellation of the formal series, which is the unit in the decomposition of $f_\beta(z)$ by the Weierstrass preparation theorem. Examples of different types of numeration algebraic bases are evoked to examplify some cases.