Maximal pattern complexity, dual system and pattern recognition

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Abstract

For a family $\Omega$ of sets in $\mathbb{R}^2$ and a finite subset $S$ of $\mathbb{R}^2$, let $p_\Omega(S)$ be the number of distinct sets of the form $S \cap \omega$ for all $\omega \in \Omega$. The maximum pattern complexity $p^*_\Omega(k)$ is the maximum of $p_\Omega(S)$ among $S$ with $\#S = k$. The $S$ attaining the maximum is considered as the most effective sampling to distinguish the sets in $\Omega$. We obtain the exact values or at least the order of $p^*_\Omega(k)$ in $k$ for various classes $\Omega$. We also discuss the dual problem in the case that $\#\Omega = \infty$, that is, consider the partition of $\mathbb{R}^2$ generated by a finite family $T \subseteq \Omega$. The number of elements in the partition is written as $p_{\mathbb{R}^2}(T)$ and $p^*_{\mathbb{R}^2}(k)$ is the maximum of $p_{\mathbb{R}^2}(T)$ among $T$ with $\#T = k$. Here, $p^*_\Omega(k) = p^*_{\mathbb{R}^2}(k)$ does not hold in general.

For the general setting that $\Omega$ is an infinite subset of $A^\Sigma$, where $A$ is a finite alphabet, $\Sigma$ is an arbitrary infinite set, and $p_\Omega(k) = \max_{\#S=k} \#\Omega|S$, it is known that the entropy

$$h(\Omega) := \lim_{k \to \infty} \frac{\log p^*_\Omega(k)}{k}$$

exists and takes value in $\{\log 1, \log 2, \cdots, \log \#A\}$. In this paper, we prove that the entropy $h(\Sigma)$ of the dual system coincides with $h(\Omega)$. 

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